On the Optimal Mix of Corporate Hedging Instruments: Linear Versus Nonlinear Derivatives

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We examine how corporations should choose their optimal mix of linear and nonlinear derivatives. We present a model in which a firm facing both quantity (output) and price (market) risk maximizes its expected profits when subjected to financial distress costs. The optimal hedging position generally is comprised of linear contracts, but as the levels of quantity and price-risk increase, the use of linear contracts will decline due to the risks associated with overhedging. At the same time, a substitution effect occurs

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toward the use of nonlinear contracts. The degree of substitution will depend on the correlation between output levels and prices. Our model also allows us to provide insight into the relation between a firm's derivatives usage and its transaction-cost structure. Jrl Fut Mark 23:217–239, 2003

INTRODUCTION

A rich literature has emerged that explores the various channels through which hedging can contribute to higher firm value.[1] Whereas these studies have increased our understanding as to why or what motivates corporations to manage risk, less attention has been directed as to how corporations should hedge, particularly, how they should choose their optimal mix of linear and nonlinear derivative instruments. Thus, the purpose of this study is to provide insight as to how corporate risk managers should construct their optimal hedging portfolios through combinations of linear (e.g., swaps, forwards, and futures) and nonlinear (e.g., options, caps, floors, and swaptions) instruments.[2]

We present a simple hedging model of a firm that faces both price (or market) risk and quantity (or output) risk, and whose objective is to maximize expected profits when subject to financial distress costs. We analyze the sensitivities of the resulting optimal linear and nonlinear hedging positions to the levels of output and price risk, the correlation between output and prices, and to the level of the fixed and variable transaction costs associated with conducting a hedging program.

Our central thesis is that when firms face only price risk, the optimal hedging position will be comprised strictly of linear instruments (e.g., forwards). This strategy can eliminate financial-distress costs resulting from low price states. However, as quantity risks become of concern, nonlinear instruments (e.g., the purchase of puts) will be substituted in part for linear instruments due to the increased likelihood that the firm will experience overhedging costs. This "overhedging problem," which is largely overlooked in the risk-management literature, results from the firm having sold too many forward contracts relative to realized output, concurrent with high realized prices. If the firm overhedges and prices rise, the firm suffers not only a revenue shortfall due to the lower output, but also finds that its revenue windfall due to higher

[1]See Smithson (1998) for an excellent summary and also the seminal articles of DeMarzo and Duffie (1995), Froot, Scharfstein, and Stein (1993), and Smith and Stulz (1985).
[2]A professional version of this article, with anecdotal and empirical support for our hypotheses, appears in Gay, Nam, and Turac (2002).

prices is more than offset by the losses on its excess forward contract position. In contrast, when output is low and prices rise, puts will finish out of the money and, beyond the option premium, will not exacerbate further distress costs. Thus, as the risk of overhedging increases, the firm will reduce its linear position; however, to maintain protection against falling prices, the firm will increase its use of nonlinear contracts. The optimal combined linear and nonlinear position minimizes the sum of the overhedging costs in high-price states and the distress costs in low-price states.

The degree of substitution between linear and nonlinear instruments is influenced strongly by the correlation between output and prices. With a negative correlation, prices likely will be high (low) when the firm's realized output is low (high). This produces a natural hedging effect and thus reduces the firm's overall demand for hedging instruments. However, a negative correlation increases the likelihood that a firm will face the problem of overhedging. In response, the firm will reduce further its linear position and substitute nonlinear contracts.

With positive correlation, prices now more likely will be low in those states where a firm has overhedged (realized low output). The firm's overall demand for derivatives will increase because, in addition to reducing price risk, derivatives can be used to reduce a portion of the firm's quantity risk. A positive correlation also will help mitigate a firm's potential overhedging problem associated with using linear contracts.

We demonstrate the separate effects that variable and fixed transaction costs associated with conducting a hedging program have on a firm's optimal hedging position. The larger the variable transaction cost, the lower the overall demand for both linear and nonlinear contracts. However, variable costs have a disproportionately larger effect on reducing the nonlinear position as compared to the linear position. Fixed transaction costs have less of an effect on the composition of the firm's optimal hedging position, but rather are an important determinant of whether the firm will undergo hedging.

PRIOR RESEARCH ON THE LINEAR/ NONLINEAR INSTRUMENT CHOICE

Early comparisons of linear and nonlinear instruments are presented in Black (1976) and Moriarty, Phillips, and Tosini (1981). These studies note the ability of linear derivatives such as futures and forwards to inexpensively transfer risk and to reduce cash flow volatility. Instruments with nonlinear payoffs such as options can reduce downside risk while allowing upside potential and can be used for yield or income enhancement. Bookstaber and Clarke (1983, p. 132) noted that, "Futures contracts will not be the appropriate hedging instrument if there is a desire to maintain a profit potential from favorable price changes or if there is uncertainty about the amount of quantity that will be held [quantity risk]. . . . Options are the instrument to overcome these shortcomings." In support, Block and Gallagher (1986) provided survey evidence that futures are perceived by managers as being advantageous in terms of their cost and efficiency of hedging whereas options are seen as having fewer administrative problems and being more effective in protecting against contingent events.

Several authors model the firm's hedging decision using an expected utility framework. Detemple and Adler (1988) considered risk-averse managers with limited access to financing and who face both price and quantity risk, but did not allow for the simultaneous choice of linear and nonlinear instruments. They predicted that firms facing borrowing constraints and higher price risk will be more active users of options. Tufano (1996), applying this model to gold-mining firms, tested whether firms facing greater financial constraints are more likely to use options, but found little supporting evidence.

The agricultural risk-management literature provides important early insights into the choice of linear and nonlinear instruments. Lapan, Moschini, and Hanson (1991) proposed a one-period model wherein utility-maximizing managers face price risk (but not production uncertainty) and choose among both forwards and options when making their hedging decisions. Assuming normally distributed prices (which allow for negative prices), they showed that the optimal hedging position will consist only of forward contracts as options become redundant. Lence, Sakong, and Hayes (1994) extended this model into a multiperiod framework and found an important hedging role for options. Sakong, Hayes, and Hallam (1993) also extended the model of Lapan et al. (1991) by allowing for production uncertainty. They found that the optimal hedging position usually will include options, in addition to forward contracts.[3]

Among studies departing from the expected utility framework, Froot et al. (1993) proposed a model wherein managers facing a single source

[3]More recently, related research in the agricultural risk-management literature has looked at innovations in agricultural output insurance markets such as revenue insurance products. For example, Hennessy (2002) analyzed the substitutability between price futures contracts and revenue futures contracts and explored issues pertaining to hedging performance, market completeness, and the resulting policy implications.

of "hedgeable" risk attempt to maximize firm value. They demonstrated that the optimal choice of hedging instruments is dependent on the relative sensitivities of internally generated cash flows and investment opportunities to changes in market prices. If the sensitivities are similar, a linear strategy (futures) will be optimal; otherwise firms may prefer options.

Brown and Toft (2002) modeled a profit-maximizing firm facing both price and quantity risk. Allowing for financial distress costs, they derived optimal hedging positions using forwards only and then options only. They found that when quantity risk is higher, or when prices and quantity are correlated negatively, put options can be superior to selling forward contracts. They also introduced a third theoretical hedging instrument referred to as a custom exotic derivative. This product, though typically non-existent in practice, is shown to be superior to either forwards or options in terms of hedging efficiency.

Mello and Parsons (2000) developed a model wherein value-maximizing managers attempt to mitigate financial-distress costs caused by firm illiquidity. They incorporated both hedgeable output and non-hedgeable input risk and, allowing only for the use of short-term futures, they derived optimal hedging positions. Adams (2001) extended this model and that of Froot et al. (1993) by focusing on the cost differential between internal and external funds. When the cost of external financing is relatively low, he found the optimal payoff to be convex, suggesting the firm's need to purchase put options. For costly external financing, the payoff will be concave, suggesting the writing of calls, whereas for intermediate cost differentials, the optimal hedging strategy will contain both elements (e.g., a collar strategy).

A HEDGING MODEL WITH QUANTITY AND PRICE RISK

The Firm's Profit Function

We consider a firm's short-run or one-period-ahead hedging problem. As in Brown and Toft (2002), the firm's investment and operating structure is predetermined. While in the long run these can be adjusted, these policies are somewhat inflexible and may indeed also be a contributing factor to distress costs in the short run. End-of-period revenues are subject to both output and market price risk. We let Z and ε denote the firm's output and market price, respectively, which together produce revenues of $Z * \varepsilon$.[4] We let π_0 represent the unhedged profit to equity holders (which we henceforth refer to as the firm's unhedged profits) as given by:

$$\pi_0(Z, \varepsilon) = Z * \varepsilon - Dr - C \tag{1}$$

where Z and ε are bivariate lognormally distributed variables having a joint probability density $g(Z, \varepsilon)$ and correlation ρ; D is the amount of firm debt outstanding; r is the fixed rate of interest on debt; and C represents the fixed dollar costs of production plus depreciation.[5] Z has an expected value of μ and standard deviation of σ, whereas ε has an expected value of ε_F and standard deviation of σ_ε. The firm's revenues are thus subject to both price risk (σ_ε) and quantity risk (σ).

The firm has access to forward and option (put) contracts written on the risky commodity with both contracts expiring at the end of the period. We assume that the forward price, ε_F, equals the future expected spot price, that is, $\varepsilon_F = E[\varepsilon]$. For simplicity, only one strike price for put options is considered; it is set to equal the forward price (or rate).

The firm chooses its forward and option positions such that it maximizes its end-of-period expected profit. At the beginning of the period, the firm sells X one-period forward contracts at a forward rate ε_F. At period end, the firm settles the X forward contracts at the then prevailing spot rate ε, producing a payoff equal to $(\varepsilon_F - \varepsilon)X$. Similarly, the firm purchases Y puts with strike price ε_F for a total end-of-period cost of PY where P is equal to the put's expected payoff. At period end, the put will have a payoff of either zero, if ε exceeds the strike price ε_F, or a gain of $(\varepsilon_F - \varepsilon)Y$ otherwise. Thus, the firm's (i.e., equity holders') hedged end-of-period profit is expressed as

$$\begin{aligned} \pi &= \pi_0(Z, \varepsilon) + (\varepsilon_F - \varepsilon)X + (\varepsilon_F - \varepsilon)lY - PY \\ &= Z * \varepsilon - Dr - C + (\varepsilon_F - \varepsilon)X + (\varepsilon_F - \varepsilon)lY - PY \end{aligned} \tag{2}$$

where $l = 1$ if $\varepsilon \le \varepsilon_F$, and $l = 0$ if $\varepsilon > \varepsilon_F$.

The Financially Unconstrained Firm

We first consider the hedging strategy of the firm that faces no financial constraints, that is, the firm is assumed to have access to unlimited

[4] Z can be viewed as the output of a commodity-producing firm (e.g., barrels of oil) that is to be sold at an uncertain spot price ε, or, alternatively, as a firm's foreign currency denominated revenues whose domestic value is subject to an uncertain spot exchange rate.

[5] We assume that the repayment of the debt principal is to occur at a later date, thus allowing our focus to be on the firm's one-period-ahead hedging problem.

risk-free borrowing. At the beginning of the period, the firm chooses the optimal mix of linear and nonlinear instruments, X^* and Y^*, respectively, that maximizes its expected end-of-period profit:

$$\max_{X,Y} E[\pi] \tag{3}$$

In the absence of financial constraints, there is no advantage to hedging as the firm's expected hedged profit is equal to its expected unhedged profit, $E[\pi] = E[\pi_0]$. Thus, $E[\pi]$ is independent of the choice of X and Y.

The Financially Constrained Firm

When profits satisfy the interest coverage or times-interest-earned ratio (TIE), as required by creditors, the firm incurs no financial distress costs. This is expressed as

$$TIE = \frac{\text{Operating Profit}}{\text{Interest Expense}} = \frac{\pi + Dr}{Dr} > \beta$$

which can be rewritten as $\pi > (\beta - 1)Dr$ where β is the specified threshold. When profits fall below this point, the firm incurs a distress cost that is proportional to the extent of the shortfall, $\gamma[\pi - (\beta - 1)Dr]^2$, where γ is a proportionality coefficient with a range of values $0 < \gamma < 1$.[6] In practice, these costs may entail higher future borrowing costs (see Diamond, 1984) or business disruption costs (see Titman, 1984).

Based on the above, managers maximize the following profit function, π_{DC}, which is equal to hedged profits less any financial distress costs (DC) incurred by equity holders:

$$\pi_{DC} = \pi - DC \tag{4}$$

where

$$\begin{aligned} DC &= 0 && \text{if } \pi \geq (\beta - 1)Dr \\ &= \gamma[\pi - (\beta - 1)Dr]^2 && \text{if } \pi < (\beta - 1)Dr \end{aligned} \tag{5}$$

To find the optimal notional amount of linear and nonlinear hedging instruments, X^* and Y^*, the following maximization of the expected

[6]Various market imperfections, including financial distress costs, a progressive tax schedule, and costly external financing, have been shown to induce concavity into the firm's value function, a necessary condition for a firm's hedging activities to be value-enhancing. See, for example Froot, Scharfstein, and Stein (1993), Géczy, Minton and Schrand (1997), Mayers and Smith (1982), and Smith and Stulz (1985).

profit of the firm is solved:

$$\max_{X,Y} E[\pi_{DC}] \tag{6}$$

Alternatively, the solution to this problem is equivalent to that obtained from the minimization of the expected distress costs as shown by the following:

$$\max_{X,Y} E[\pi_{DC}] = \max_{X,Y} E[\pi - DC] = \max_{X,Y} (E[\pi] - E[DC])$$
$$= E[\pi_0] - \min_{X,Y} E[DC]$$

We thus solve the following minimization problem:

$$\min_{X,Y} E[DC] \tag{7}$$

To minimize the expected distress cost function, one must integrate over both random variables Z and ε. However, after integrating over variable Z, we obtain an expression that cannot be further analytically integrated over the variable ε due to discontinuity. The interested reader can find this resulting expression in the Appendix, denoted as A1. Therefore, to find the optimal linear (X^*) and nonlinear (Y^*) positions, the solution to Expression A1 is found numerically.

SENSITIVITY ANALYSIS OF THE GENERAL SOLUTION

Quantity and Price-Risk Effects

To illustrate the effect of quantity and price risk on the optimal mix of linear (X^*) and nonlinear (Y^*) instruments, we use the following base case-parameter values in arriving at our numerical solutions. We assume expected revenues are equal to \$10 based on a level of expected output (μ) of 10 units and an expected price of the commodity produced (ε_F) of \$1 (or which is analogous to the firm having expected foreign currency revenues of 10 FC and an expected exchange rate of \$1/FC). We also assume a face value of debt (D) equal to \$10; an interest rate (r) equal to 10%; and a level of fixed production costs (C) equal to \$6. Thus, the interest payment rD is equal to \$1, the expected unhedged profits $E[\pi_0]$ are equal to 3, and the expected times-interest-earned (TIE) ratio is equal to 4. In addition, the proportionality cost coefficient γ is equal to 0.1, and the firm incurs financial distress costs when the threshold β for TIE is less than 2.

Figure 1 provides the first set of results under the initial assumption of a zero correlation between output and prices (i.e., $\rho_{Z,\varepsilon} = 0$). The

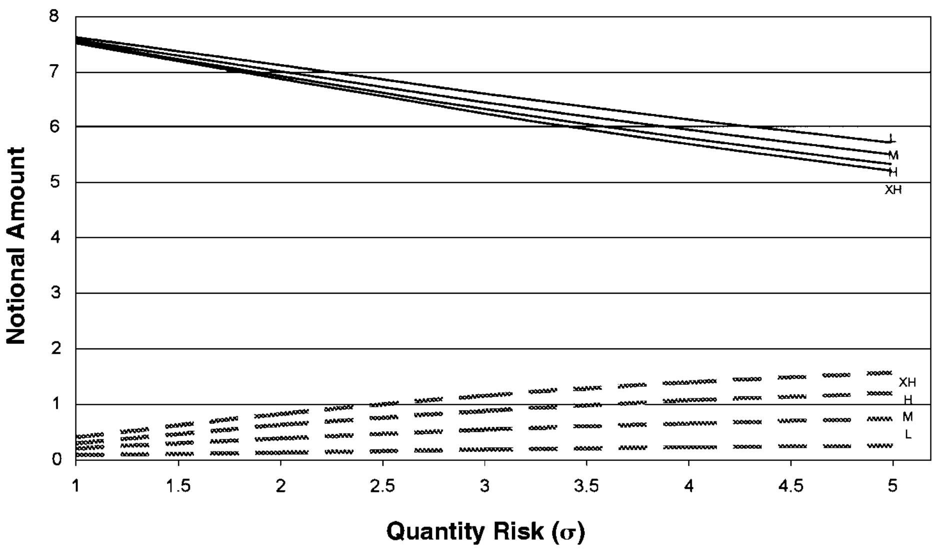

FIGURE 1

Optimal combinations of linear and nonlinear derivative instruments as a function of quantity and price risk. Solid lines in the figure represent the optimal notional amounts of linear instruments as a function of quantity risk (measured as the standard deviation of the firm's expected output) for different levels of price risk (measured as the standard deviation of the output market price). Similarly, the dashed lines represent the optimal notional amounts of nonlinear instruments. In the figure, the expected output level and forward output price are \$10 and \$1, respectively. The coefficient of correlation, $\rho_{Z,\varepsilon}$, between output and prices is assumed to be 0. The firm's fixed dollar costs of production are \$6 and interest expenses are \$1. The various levels of price risk are denoted by *L*, *M*, *H*, and *XH* (low, medium, high, and extra high) and correspond to price-risk standard deviations of 0.1, 0.3, 0.5, and 0.7, respectively.

optimal notional amount of both linear (X^*) and nonlinear (Y^*) contracts is plotted along the vertical axis whereas the level of quantity risk (σ) is plotted along the horizontal axis. For each level of quantity risk, we report results for values of price risk (σ_ε) equal to 10%, 30%, 50%, and 70%.

We note several observations on the level and pattern of the optimal hedging positions. First, for low and moderate levels of output risk, a firm's risk exposure is optimally hedged mainly using linear or forward contracts (represented by the solid curves in Figure 1), with a relatively minor position in nonlinear or option contracts (represented by the dashed curves). Second, as the level of output risk increases, the optimal linear position declines whereas the nonlinear position increases. Third, holding constant the level of output risk, the optimal linear position is decreasing in price risk, that is, the greater the level of price risk, the smaller the use of forward contracts. In contrast, the optimal nonlinear position is increasing in market risk.

It appears that under "normal" market and business operating conditions, firms should conduct their hedging primarily using linear contracts. Furthermore, the optimal linear position will be to sell a notional amount of forward contracts that is somewhat below the value of expected output. That is, the firm will engage in less than full hedging. In addition, as firms face increasingly higher levels of output risk, they will reduce further their use of linear contracts, but at the same time use more nonlinear contracts. Furthermore, this substitution effect becomes greater the higher the level of price risk.

Discussion

Hedging in the Absence of Quantity Risk

Linear Contracts. Using the results observed in the above numerical exercise as a basis for discussion, consider first the firm's hedging decision regarding linear contracts under zero-output risk. The hedging decision is relatively straightforward. The optimal linear strategy is for the firm to sell forward a quantity of its production that will ensure a minimum guaranteed level of profit so as to avoid the triggering of financial distress costs.

As shown in Expression 5, the firm avoids triggering financial-distress costs when its hedged profits π are sufficiently high enough to satisfy coverage ratio requirements, that is, $\pi \geq (\beta - 1)Dr$. Substituting for π from Equation 2 into this condition, the firm will sell a quantity of forward contracts X that ensures a sufficient level of hedged revenue (i.e., revenues from selling output plus hedging gains and losses) to cover the sum of its fixed production costs $C + \beta * Dr$. That is, the firm will choose X such that $Z * \varepsilon + (\varepsilon_F - \varepsilon) * X$ will be at least $C + \beta * Dr$. Using the base case assumptions of $Z = 10$, $\beta = 2$, interest expense $(Dr) = \$1$, $\varepsilon_F = \$1$, and fixed production costs $(C) = \$6$, the firm requires a minimum level of hedged revenues of \$8. To ensure such a level of revenue, even in the worst case that the realized price of output ε falls to zero, the firm will sell a notional quantity of 8 forward contracts. This corresponds to the optimal linear position depicted in Figure 1, when quantity risk approaches zero.

Although popular belief often holds that a firm's hedge position should match expected output (or foreign revenues), the above illustration makes the similar observation as in Mello and Parsons (2000) that the firm's optimal hedging position typically will be to partially hedge. The hedge position needs only to ensure that the firm is able to generate a sufficient level of revenue to avoid distress costs. In our example, although the firm's entire output of 10 units is subject to price risk, the firm can hedge as few as 8 units of output and still avoid financial-distress costs. Ignoring transaction costs, the firm actually is indifferent to hedging any quantity of units between 8 and 10, but hedging additional units beyond 8 will produce zero expected net benefits.

Hedging beyond 10 contracts, however, exposes the firm to costs related to "overhedging." We loosely define overhedging as having determined ex post that an excessive quantity of forward contracts was sold relative to realized output. Thus, hedging more than 10 units is suboptimal because the additional forward positions will increase expected distress costs. Conditioned on the firm already hedging at least 8 units, gains on additional short forward positions when prices fall cannot further reduce distress costs because they have been eliminated. However, if prices rise, the firm's loss on the additional forward contracts will exceed the extra revenues generated from selling output at higher prices. If prices rise significantly, the firm's hedged revenues could fall below a level that triggers financial-distress costs. We refer to this increase in expected financial-distress costs due to hedging with an excessive number of linear contracts as the "cost of overhedging."

Nonlinear Contracts. Generally speaking, at the margin, the use of nonlinear instruments will likely have negligible (or even negative) benefits. That is, conditioned on the firm being at its optimal linear position, substituting nonlinear for linear contracts may not be attractive due to (1) the relatively greater number of contracts needed (thus, likely higher transaction costs), and (2) limits on the extent of their use because of the potential for the payment of option premiums to trigger financial-distress costs.

To see this, recall that the firm's optimal linear-hedging position was 8 contracts. (Although 9 and 10 contract positions also were optimal, the 9th and 10th contracts were superfluous in terms of contributing to firm value.) Consider first the substitution of a long put for the last (i.e., 8th) forward contract. The firm's hedging position will now consist of 7 short forward contracts and one long put contract with an exercise price of \$1. Due to the option premium, the put will not provide the same level of net downside protection against potential distress costs, as did the forward contract. To illustrate, assume a put value of \$0.20 and that output prices fall to zero. The firm's revenues on output ($Z * \varepsilon$) will be zero, the profits on the 7 forward contracts will be \$7 and the net profit on the put will be (\$1–\$0.20) or \$0.80, together producing a total hedged revenue of only \$7.80. Thus, the firms will incur financial-distress costs as this number is less than the \$8 ($C + \beta Dr$). To avoid these costs, the firm should instead purchase options in the ratio of \$1/(\$1–\$0.20) or 1.25 puts per forward contract. In terms of hedging performance, the firm is indifferent between using either the 1.25 puts or 1 forward contract. However, if transaction costs are comparable, the forward contract will be preferred due to the lower quantity required.

Hedging in the Presence of Quantity Risk

Linear Contracts. When quantity risk becomes non-zero, the uncertainty regarding actual output levels makes avoiding the costs of overhedging a more difficult task. Consider the situation where expected output remains at 10 units, but now because of quantity risk, the firm's actual output can vary from 6 to 14 units. Previously, 8 forward contracts were optimal. If the firm continues with an 8 forward contract position, then overhedging can occur if realized output falls between 6 and 8 units. To avoid completely the risk of overhedging, the firm would sell 6 or less forward contracts. However, selling less than 8 contracts exposes the firm to potential additional distress costs in the event that prices fall. Thus, the firm will use somewhere between 6 and 8 contracts. Generally speaking, the optimal linear position will be such that it minimizes the sum of the expected distress costs from falling prices and the costs of overhedging from rising prices (henceforth, "total overall expected distress costs"). This also is to say that at the optimum, the marginal expected distress and overhedging costs from adding or subtracting contracts are equal.

Nonlinear Contracts. There is now a limited, but important role for nonlinear contracts because of their ability to fine-tune the optimal risk-management strategy to reduce further the sum of the expected distress and overhedging costs. Given the optimal linear position, consider first the effect of the substitution of a long put in place of the last short forward contract.[7] Compared to the foregone forward contract, the put is less effective for reducing expected distress costs in low price states due to the option premium. However, in high-price states, the put will be more advantageous as the put will expire worthless and with its net payoff limited to the loss of the option premium. Thus, the substitution of puts for forward contracts can alter the total expected distress costs in a positive or negative manner. The substitution will be optimal if the

[7]Substituting puts for forwards in this fashion can be thought of as somewhat analogous to buying calls. That is, buying a put and closing out or buying back a short forward contract can be replicated by buying a call.

decrease in overhedging costs exceeds the increase in expected distress costs, thus reducing the total overall expected distress costs. However, if the substitution increases the overall costs, the firm instead should engage in "reverse substitution" wherein they simultaneously add additional short forward contracts and write puts (which together can be viewed as equivalent to "writing calls").

The extent of substitution in either case, and hence the extent of using options is, however, limited. To see this, recall that the optimal linear position in the previous example was said to be between 6 and 8 contracts. Assume that 7 linear contracts is the optimal position. Under zero correlation, if either substitution or reverse substitution is optimal, the resulting optimal linear positions will never fall below 6 contracts (a reduction of one forward) or above 8 contracts (an addition of one forward). At a level of 6 forward contracts, all costs of overhedging are eliminated, whereas at 8 contracts, the expected distress costs in low-price states cannot be reduced further through additional forwards. Thus, based on a 1.25 substitution rate, the potential maximum number of puts will be limited to only 1.25 contracts (long or short).

As output risk increases, there will be a greater opportunity for substitution, and hence a larger role for options. To illustrate, assume that output can range from 4 to 16 units. Thus the optimal linear position will be somewhere between 4 to 8 contracts. Assume that it is 6 forward contracts. Depending on whether substitution or reverse substitution is optimal, as many as 2 forwards could be eliminated or added and, therefore, as many as 2.5 long or short put contracts established.

Finally, there is one additional consideration. Even if prices remain relatively stable, the firm can incur financial distress if output falls significantly. Forward contracts will not provide relief in these states, but the additional income from writing puts could provide additional protection. The implication of this is that the propensity to buy put options will be reduced, thus weakening (strengthening) the substitution (reverse substitution) effect noted above.

Correlation Effects

Depending on the sign of the correlation between output and prices ($\rho_{Z,\varepsilon}$), revenues will exhibit either greater or lower volatility. A positive correlation will exacerbate fluctuations in revenues because output levels and prices will move in the same direction. The implications are twofold. First, the overall demand for derivatives will increase as the quantity risk becomes more "hedgeable" because output now moves more in line with prices.

Second, the overhedging problem becomes less severe because the likelihood of observing a simultaneous drop in output and an increase in price is diminished.

A negative correlation between output and prices will produce an opposite result, as it will serve to dampen fluctuations in revenues, thus producing a “natural hedge” effect. For example, a negative correlation for an exporting firm selling goods in a foreign country implies that it will experience increasing (declining) sales volume at the same time as the domestic currency is strengthening (weakening). Alternatively, a commodity firm experiencing higher (lower) output will have the potential increase (decrease) in revenues offset to some extent by a lower (higher) market-selling price.

To illustrate the effect of correlation, Figure 2 provides a comparison of the optimal hedging positions corresponding to values of $\rho_{Z,\varepsilon}$ equal to -0.25, 0, and $+0.25$. Panels (a) and (b) present the optimal linear and nonlinear positions, respectively, for various levels of quantity risk and a price-risk level of 30%. As shown in Panel (a), the higher the correlation, the greater the optimal linear position for a given level of quantity risk. This can be attributed to the overhedging problem becoming less (more) severe the greater (lower) the correlation. As mentioned, the overhedging problem becomes of greatest concern in states where output falls and prices rise. If quantity and prices are correlated positively, this occurrence becomes less likely, thus leading the firm to use more linear instruments.

The corresponding optimal nonlinear positions are presented in Panel (b). As shown, the substitution of nonlinear for linear contracts is related inversely to the level of correlation. The greater (lower) the correlation, the lower (greater) the nonlinear position. For negative correlation, the nonlinear position is initially both greater than that observed for zero correlation and is increasing in quantity risk. Although the position is still relatively small, the larger nonlinear position (than in the zero-correlation case) helps mitigate the overhedging problem and provides additional protection against large declines in prices that otherwise could generate significant financial-distress costs.

As correlation becomes increasingly positive, the nonlinear position declines in size and even can become negative. In this case, the firm switches from essentially purchasing put protection to a strategy of put writing (i.e., “reverse substitution”) for purposes of generating premia income to add to its revenue stream. Intuitively, to see why a firm would pursue such a strategy of writing puts, note that the increased linear position shown in Panel (a) due to the positive correlation, coupled with

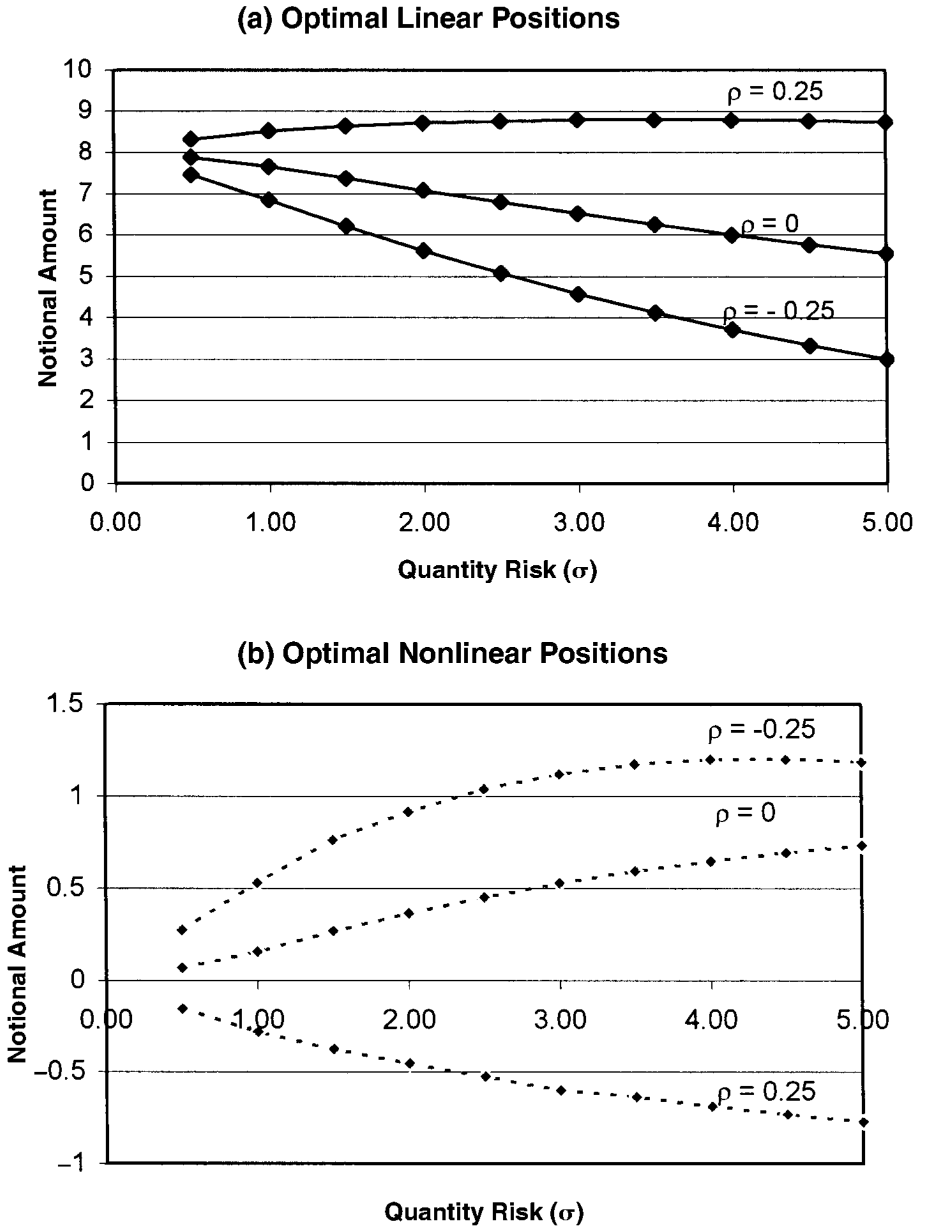

FIGURE 2

Effects of correlation on optimal linear and nonlinear positions. Solid lines in Panel (a) plot optimal notional amounts of linear instruments as a function of quantity risk (measured as the standard deviation of the firm's expected output) for different values of the coefficient of correlation, $\rho_{Z,\varepsilon}$, between output and prices. Similarly, the dashed lines in Panel (b) plot optimal notional amounts of nonlinear instruments. The coefficients of correlation chosen are +0.25, 0, and −0.25. The expected output level and forward output price are 10 and $1, respectively. Market risk, σ_ε, measured as the standard deviation of the output price is $0.30. Fixed dollar costs of production are $6 and interest expenses are $1.

put writing, is equivalent to writing calls. The revenue from writing options serves to offset the lower revenue in either low-price or low-output states, thus reducing potential financial-distress costs. In high-price states, the potential losses on the calls are of less concern because output, and hence revenue, is likely to be higher due to the positive correlation.

TRANSACTION COSTS

We next examine how transaction costs associated with supporting and executing trades of linear and nonlinear instruments affect the firm's optimal hedging strategy. To incorporate transaction costs into our analysis, we modify the firm's profit function given earlier in Expression 4 as follows:

$$\pi'_{DC} = \pi - DC - TC \tag{8}$$

where

$$TC = Abs[X]t_x + Abs[Y]t_y + T \tag{9}$$

with TC denoting the total hedging transaction costs, t_x and t_y being the variable transaction costs per unit of linear and nonlinear instruments, respectively, and T representing the fixed transaction cost. As before, the optimal quantity of linear and nonlinear positions are found numerically by solving the following maximization problem:

$$\max_{X,Y} E[\pi'_{DC}] \tag{10}$$

which is equivalent to the minimization of the expected sum of the distress and transaction costs as given by:

$$\max_{X,Y} E[DC + TC] \tag{11}$$

Variable Transaction Costs

We first inspect the effect of the variable transaction-cost rate (t_x and t_y) using the same base case assumptions as previously used and with a fixed level of quantity risk of 3. We allow the variable transaction costs to vary from \$0 to \$0.01 per unit (which is equivalent to a transaction rate ranging from 0%–1%) and initially assume that the fixed transaction costs (T) are zero. The results of the numerical solutions to Expression 11 are presented in Figure 3.

In each of the panels in Figure 3, results are presented for the three correlation values of 0.25, 0, and −0.25. Panels (a) and (b) illustrate the

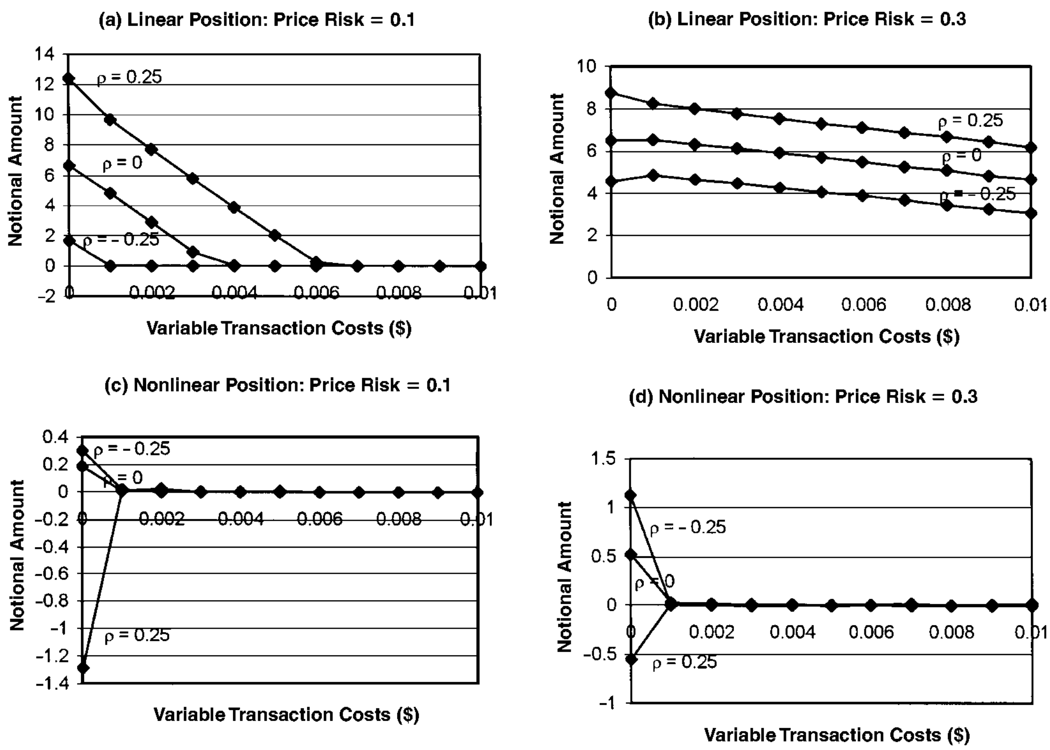

FIGURE 3

Optimal combinations of linear and nonlinear derivative instruments as a function of variable transaction costs. Panels (a) and (b) plot optimal notional amounts of linear instruments as a function of variable transaction costs and for three different values of the coefficient of correlation, $\rho_{Z,\varepsilon}$, between output and prices. Similarly, Panels (c) and (d) plot optimal notional amounts of nonlinear instruments. The coefficients of correlation between output and market prices are chosen to be +0.25, 0, and −0.25. The expected output level and forward output price are 10 and \$1, respectively. Price risk, σ_ε, measured as the standard deviation of the expected output price is 0.1 in Panels (a) and (c), and is 0.3 in Panels (b) and (d). Quantity risk, measured as the standard deviation of the firm's expected output is assumed to be 3. Fixed dollar costs of production are \$6 and interest expenses are \$1.

optimal linear positions for market-risk values of 10% and 30%, respectively, whereas Panels (c) and (d) similarly illustrate the optimal nonlinear positions for the same market-risk values. Consistent with intuition, the presence of variable transaction costs reduces the demand for both linear and nonlinear contracts. As shown in Panels (c) and (d), as the variable cost increases, the optimal nonlinear position moves rapidly toward zero. For the linear positions in Panels (a) and (b), a higher variable cost also affects the optimal position; however, the effect is not as large as it was for nonlinear contracts.

To see this, note that in the absence of transaction costs, the marginal benefit from either additional δ units of linear or nonlinear positions when at the equilibrium is equal to zero. When variable transaction costs are introduced, the net marginal benefit (the marginal benefit minus the variable transaction rate) becomes negative at the no-transaction-cost equilibrium. Thus, to reach a new equilibrium, both the optimal linear and nonlinear position will be scaled back. Because the nonlinear positions initially are relatively small for reasons discussed earlier, they will drop out earlier than linear positions. The reduction in the nonlinear position as variable costs increase can, under certain conditions, give rise to a small substitution effect affecting the linear position. As shown in Panels (b) and (d) for high market risk (.3) and for zero and negative correlation values, the long put positions are reduced as the variable transaction-cost rate increases. However, there is initially a small increase in the linear position. As the long put position is reduced, there is lower protection against low prices, and thus additional short forwards are substituted. However, once the put position is reduced to zero, the linear position also will start to decline. This substitution effect was not observed in Panels (a) and (c) because, for low levels of price risk, the reduction in expected distress costs from substituting linear contracts was smaller than the additional transaction costs.

Fixed Transaction Costs

With respect to the effect of fixed transactions costs (T), it is first important to point out that its magnitude will be a primary determinant of whether hedging will be initiated. For a given level of fixed transaction costs, there is a corresponding threshold of quantity/price-risk combinations below which hedging will not be optimal, and thus no position should be initiated. This is because, beneath this threshold, the expected reduction in financial-distress costs (i.e., the benefits from hedging) is lower than the fixed transaction costs. Beyond the threshold, the effect of the fixed transaction cost will depend on its relative size as compared to the sum of the firm's fixed dollar cost of production (C) and interest expense (Dr).

For larger firms that enjoy scale economies in hedging, the effect of fixed transaction costs on the optimal hedging position will be relatively minor. For smaller firms, the relative cost of establishing a hedging program could be significant. This could affect not only its decision to hedge, but also the optimal hedging position. This suggests that smaller firms will tend not to hedge unless they face large quantity and/or price risks, but conditioned on hedging, they should hedge relatively more than large firms having comparable relative cost structures.

In Table I we explore these effects in which we again use the base case assumptions employed in conducting the earlier analyses (along

TABLE I

Effect of Fixed Transaction Costs on the Optimal Hedging Position

	$t_x = t_y = \$0.0000$			$t_x = t_y = \$0.0002$			$t_x = t_y = \$0.0004$		
T	X^*	Y^*	$E_{DC} + T + t_x X^* + t_y Y^*$	X^*	Y^*	$E_{DC} + T + t_x X^* + t_y Y^*$	X^*	Y^*	$E_{DC} + T + t_x X^* + t_y Y^*$
$0.00	6.517	0.527	$0.066	6.596	0.260	$0.068	6.674	0.001	$0.070
$0.01	6.523	0.528	$0.077	6.600	0.264	$0.079	6.680	0.003	$0.081
$0.02	6.529	0.528	$0.088	6.606	0.265	$0.089	6.685	0.005	$0.091
$0.03	6.533	0.534	$0.099	6.613	0.269	$0.100	6.691	0.005	$0.103
$0.04	6.541	0.531	$0.109	6.618	0.271	$0.111	6.694	0.007	$0.112
$0.05	6.548	0.531	$0.120	6.623	0.274	$0.122	6.697	0.013	$0.123
$0.06	6.553	0.532	$0.131	6.628	0.276	$0.132	6.702	0.014	$0.135
$0.07	6.560	0.533	$0.141	6.635	0.277	$0.143	6.712	0.017	$0.143
$0.08	6.565	0.534	$0.152	6.641	0.277	$0.154	6.718	0.021	$0.155
$0.09	6.571	0.536	$0.163	6.645	0.281	$0.164	6.722	0.022	$0.165
$0.10	6.576	0.537	$0.174	6.652	0.283	$0.175	6.728	0.027	$0.176
$0.11	*0	*0	*$0.184	*0	*0	*$0.186	*0	*0	*$0.187
$0.12	*0	*0	*$0.195	*0	*0	*$0.197	*0	*0	*$0.198

Note. The table presents the optimal linear (X^*) and nonlinear (Y^*) hedging positions corresponding to various combinations of fixed (T) and variable (t_x and t_y) transaction costs. In addition, the table reports the corresponding level of the sum of the total expected distress costs and total transaction costs ($E_{DC} + T + t_x X^* + t_y Y^*$). For comparison purposes, the total expected distress cost in the absence of hedging is $0.1836. Thus, hedging no longer will become optimal when the sum of the total expected distress costs and total transaction costs associated with a given hedging position is in excess of $0.1836. The firm's assumed parameter values are as follows: expected revenue equal to $10, interest expense ($Dr$) equal to $1, fixed dollar costs of production equal to $6, quantity risk ($\sigma$) equal to 3, market risk (σ_e) equal to 0.3, and the correlation between output and prices (ρ) equal to 0.

*Hedging is no longer optimal because the sum of the total expected distress costs and total transaction costs ($E_{DC} + T + t_x X^* + t_y Y^*$) is greater than the total expected distress costs under no hedging ($0.1836).

with a zero-correlation assumption). Based on these parameters, for reference purposes, the firm's total overall expected distress costs in the absence of any hedging are equal to \$0.1836. Thus, for hedging to be justified, the resulting total distress costs after hedging plus the sum of the variable and fixed transactions costs should be less than \$0.1836. As shown in the first row of Table I, for fixed transaction costs equal to zero (T = \$0) and variable transaction costs equal to zero ($t_x = t_y$ = \$0), the optimal hedging position consists of 6.517 linear contracts and 0.527 nonlinear contracts. This result also can be discerned in Panels (b) and (d) of Figure 3. Hedging under these parameter values is justified, as the sum of the total expected distress costs and transaction costs ($E_{DC} + T + t_xX^* + t_yY^*$) reported in the third column is \$0.066 (which is less than \$0.1836). Holding constant the variable transaction cost at \$0.00, if fixed transaction costs are increased from T = \$0 to T = \$0.10 (for perspective, to approximately 1.43% of the sum of the fixed production costs C and the fixed interest expense Dr), the hedging position changes slightly to 6.576 linear contracts and 0.537 nonlinear contracts and hedging would remain optimal as the sum of the total distress costs plus transaction costs equals \$0.174. However, if the fixed transaction costs are increased further to \$0.12, then the benefits from hedging are negated as the sum of the total distress and transactions costs becomes \$0.184. Thus, the firm will no longer hedge. Finally, we note that for a given level of variable transaction costs and, conditioned on hedging being optimal for the firm to undergo, the linear and nonlinear positions are relatively insensitive to the level of fixed transaction costs.

CONCLUSION

This article adds to the literature on financial risk management by investigating how firms should choose the optimal mix of linear and nonlinear derivative instruments and the factors influencing such decisions. Our analysis indicates that linear instruments typically will dominate a firm's hedging mix, but that this usage of linear products will decline the greater a firm's quantity risk and the greater the price risk related to their output. In addition, we find that the use of nonlinear instruments typically will be increasing in both quantity and price risk. The degree of the substitution effect between linear and nonlinear instruments will be influenced strongly by the level of correlation between the firm's output and prices. We also show that the fixed transaction costs associated with initiating and maintaining a hedging program will not have a large effect on the optimal hedging position, per se, but rather will have a stronger effect on the decision to hedge. In contrast, variable transaction costs will have a relatively larger effect on the hedging positions, particularly with respect to the optimal nonlinear position. These latter findings regarding transaction costs suggest that empirical investigations of corporate derivatives use may wish to disaggregate and separately analyze firms' linear and nonlinear positions and to control for differences in firms' cost structures.

APPENDIX

To find the optimal linear and nonlinear positions, X^* and Y^*, we solve the following minimization problem:

$$\min_{X,Y} E[DC] \tag{7}$$

The solution to this problem requires integration over both random variables Z and ε, which we assume have the following bivariate lognormal distribution function:

$$g(Z,\varepsilon) = \frac{1}{\sqrt{2\pi\sigma_L^2}\sqrt{2\pi\sigma_{\varepsilon L}^2}\sqrt{1-\rho^2}Z\varepsilon} \times \exp\left\{\frac{-1}{2(1-\rho^2)}\left[\frac{(\ln Z-\mu_L)^2}{\sigma_L^2} - 2\rho\frac{(\ln Z-\mu_L)}{\sigma_L}\frac{(\ln\varepsilon-\varepsilon_{FL})}{\sigma_{\varepsilon L}} + \frac{(\ln\varepsilon-\varepsilon_{FL})^2}{\sigma_{\varepsilon L}^2}\right]\right\}$$

In the above expression, the parameters μ_L, ε_{FL}, σ_L, and $\sigma_{\varepsilon L}$ are the drift rates and volatilities of variables Z and ε, respectively, and correspond to the desired expected values μ and ε_F and standard deviations σ and σ_ε of variables Z and ε, respectively. They are related as follows:

$$\sigma_{\varepsilon L} = \sqrt{\ln\left[\left(\frac{\sigma_\varepsilon}{\varepsilon_F}\right)^2 - 1\right]}, \quad \varepsilon_{FL} = \ln\varepsilon_F - \sigma_{\varepsilon L}^2/2,$$

$$\sigma_L = \sqrt{\ln\left[\left(\frac{\sigma}{\mu}\right)^2 - 1\right]}, \quad \mu_L = \ln\mu - \sigma_L^2/2$$

After integrating over variable Z, however, we obtain the following expression that cannot be further analytically integrated over the variable ε:

$$\min_{X,Y}\frac{1}{\sqrt{2\pi}\sigma_{\varepsilon L}}\int_0^\infty \frac{d\varepsilon}{\varepsilon} e^{\{-(\ln\varepsilon-\varepsilon_{FL})^2\}/2\sigma_{\varepsilon L}^2} \times\left[\gamma\sum_{n=0}^{2}[g_n\varepsilon^{n(\phi+1)}(k-W\varepsilon)^{2-n}N(Z_1(\varepsilon)-n\sigma_L\sqrt{1-\rho^2})] + KN(Z_2(\varepsilon))\right] \tag{A1}$$

where

$$g_n = \exp\{n[\mu_L + n\sigma_L^2(1-\rho^2) - \varepsilon_{FL}\phi]\}, \quad n = 0, 1, 2$$

$$Z_i(\varepsilon) = \frac{\ln(Z^i_{\max}) - \mu_L - \phi(\ln\varepsilon - \varepsilon_{FL})}{\sigma_L\sqrt{1-\rho^2}}, \quad i = 1, 2$$

$$Z^i_{\max} = \mathrm{Max}[0, W - k_i/\varepsilon], \quad i = 1, 2$$

$$W = X + \mathrm{Max}[0, Y]$$

$$k_1 = k - (\beta - 1)Dr$$

$$k_2 = k + NWC$$

$$k = -Dr - C + \varepsilon_F(X + \mathrm{Max}[0, Y]) + PY$$

$$\phi = \rho\frac{\sigma_L}{\sigma_{\varepsilon L}}$$

$N(\cdot)$ is cumulative normal distribution function with mean zero and unit variance. We therefore solve Expression A1 numerically in order to find the optimal linear and nonlinear positions.

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